

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

ATKINSON SCIENCE

$$e^{i\pi} = -1$$
$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln \frac{y u_\tau}{\nu}$$
$$E_b = \sigma T^4$$

USER GUIDE

Oblique Shock Web Application

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Atkinson Science welcomes your comments on this User Guide. Please send an email to keith.atkinson@atkinsonscience.co.uk.

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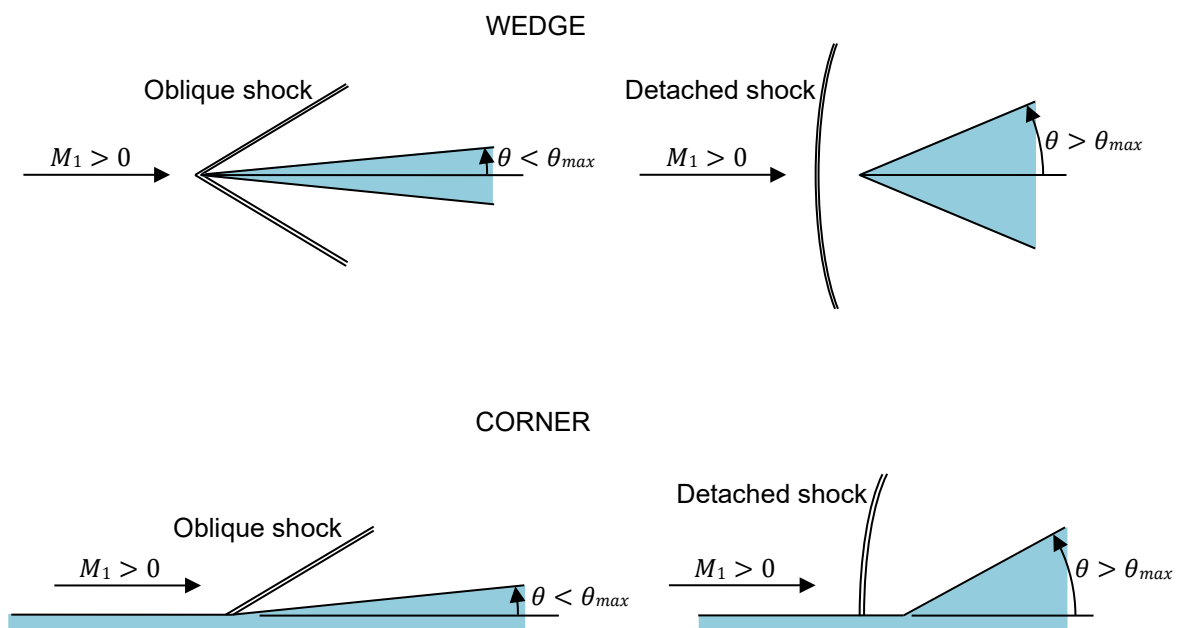
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1 Introduction

You can find the Atkinson Science Oblique Shock web application at the web page <https://atkinsonscience.co.uk/WebApps/Aerospace/ObliqueShock.aspx>. There is also a theory guide that you can download from the same page. The application calculates the change in properties across a steady, two-dimensional oblique shock. It was developed for use with the International Standard Atmosphere, Ref. [1], for which the ratio of specific heats γ is defined to be 1.4. The application was created in Microsoft Visual Studio 2017.

An oblique shock is formed when a supersonic flow approaches a wedge or corner, as shown in Figure 1. The equations for the oblique shock show that if the deflection angle θ is less than θ_{max} , where θ_{max} is dependent on the upstream Mach number, then there are two possible oblique shock solutions: a weak shock and a strong shock. In both cases the shock is *attached* to the wedge or corner, as shown in Figure 1. The changes in flow conditions across a strong shock are more severe than those across a weak shock. The weak shock is far more common and is often seen at the outer surfaces of supersonic aircraft. The strong shock tends to be seen only in internal flows where the flow must adjust to a high-pressure downstream condition. If $\theta > \theta_{max}$ then a *detached* shock is formed as shown in Figure 1. The oblique shock application does not deal with the detached shock and will inform the user when $\theta > \theta_{max}$.

Figure 1 Oblique (attached) shock and detached shock



2 User interface

The user interface of the web application is shown in Figure 2. The interface consists of two boxes. The upper box displays the properties of the flow upstream of the shock, and the lower box displays the properties of the flow downstream of the shock, with the properties for the weak shock in the left-hand column and the properties for the strong shock in the right-hand column. The user must enter three properties into the upper box before clicking the Calculate button to determine all of the other properties.

Figure 2 User interface

Oblique Shock

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Upstream of shock

<input checked="" type="radio"/> Geometric altitude <input style="width: 50px;" type="text"/> m <input type="radio"/> Geopotential altitude <input style="width: 50px;" type="text"/> m <input checked="" type="radio"/> Mach number <input style="width: 50px;" type="text"/> <input type="radio"/> Speed <input style="width: 50px;" type="text"/> m s ⁻¹ Deflection angle <input style="width: 50px;" type="text"/> deg <input type="button" value="Calculate"/>	Temperature <input style="width: 50px;" type="text"/> K Pressure <input style="width: 50px;" type="text"/> Pa Density <input style="width: 50px;" type="text"/> kg m ⁻³ Static enthalpy <input style="width: 50px;" type="text"/> kJ kg ⁻¹ Entropy <input style="width: 50px;" type="text"/> kJ kg ⁻¹ K ⁻¹ Speed of sound <input style="width: 50px;" type="text"/> m s ⁻¹
---	--

Weak shock	Strong shock
Mach number <input style="width: 50px;" type="text"/>	Mach number <input style="width: 50px;" type="text"/>
Speed <input style="width: 50px;" type="text"/> m s ⁻¹	Speed <input style="width: 50px;" type="text"/> m s ⁻¹
Temperature <input style="width: 50px;" type="text"/> K	Temperature <input style="width: 50px;" type="text"/> K
Pressure <input style="width: 50px;" type="text"/> Pa	Pressure <input style="width: 50px;" type="text"/> Pa
Density <input style="width: 50px;" type="text"/> kg m ⁻³	Density <input style="width: 50px;" type="text"/> kg m ⁻³
Static enthalpy <input style="width: 50px;" type="text"/> kJ kg ⁻¹	Static enthalpy <input style="width: 50px;" type="text"/> kJ kg ⁻¹
Entropy <input style="width: 50px;" type="text"/> kJ kg ⁻¹ K ⁻¹	Entropy <input style="width: 50px;" type="text"/> kJ kg ⁻¹ K ⁻¹
Speed of sound <input style="width: 50px;" type="text"/> m s ⁻¹	Speed of sound <input style="width: 50px;" type="text"/> m s ⁻¹
Wave angle <input style="width: 50px;" type="text"/> deg	Wave angle <input style="width: 50px;" type="text"/> deg

The application is intended for use with the International Standard Atmosphere, Ref. [1]. Consequently, the temperature, pressure and other thermodynamic properties of the air upstream of the shock can all be obtained easily, given the geometric or geopotential altitude. The user can choose between entering the geometric altitude or the geopotential altitude. The user must also enter the Mach number or speed of the flow and the deflection angle θ . The ratio of the specific heats γ of the International Standard Atmosphere is defined to be 1.4.

Figure 3 shows the output from the web application after the user has entered a geometric altitude of 10,000 m, a Mach number of 2.0, and a deflection angle of 15°.

Figure 3 Calculated properties

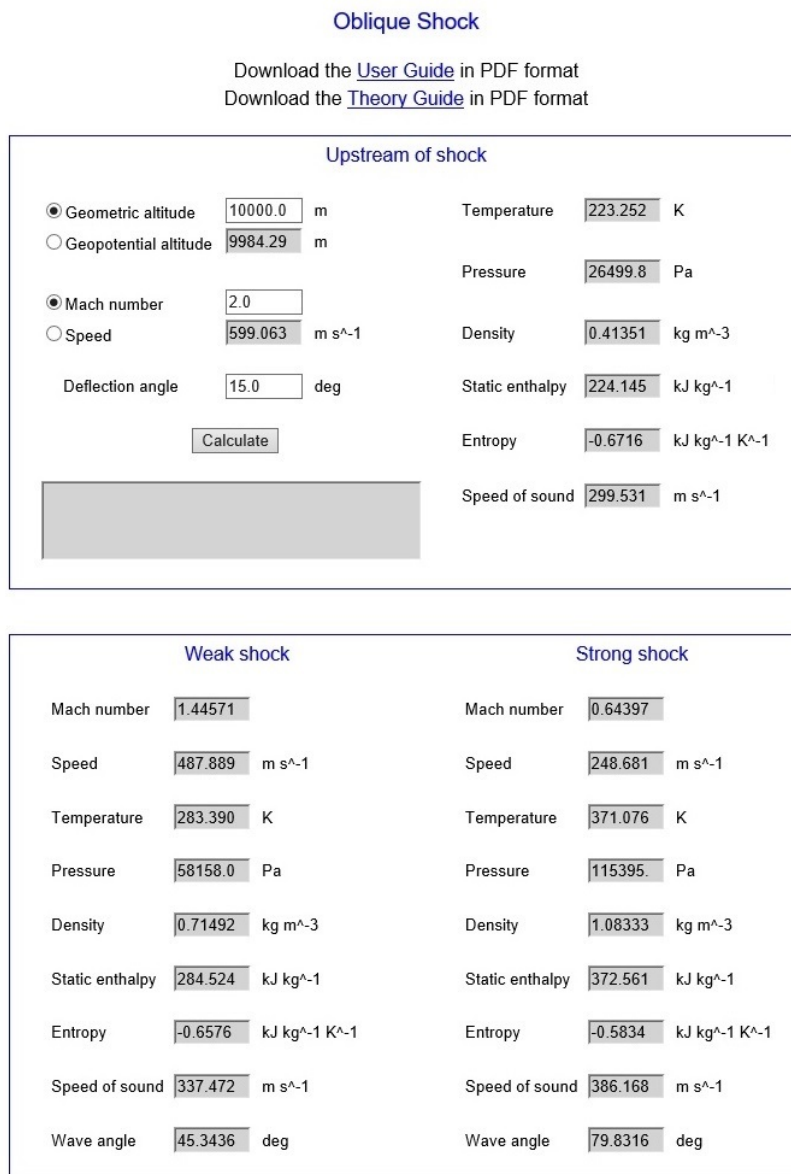


Figure 4 is a plot of all possible oblique shock solutions. The weak shock and the strong shock calculated by the web application are indicated on the plot by crosses (+). The calculated wave angles of the weak shock and the strong shock are 45.34° and 79.83°, respectively. Figure 5 shows how the shocks appear when formed at a wedge or a corner.

From Figure 4 we can see that for a Mach number of 2.0, θ_{max} is approximately 23°. If the user changes the deflection angle from 15° to, say 25°, and clicks the Calculate button, then the application will display a message as shown in Figure 6.

Figure 4 Oblique shock solutions

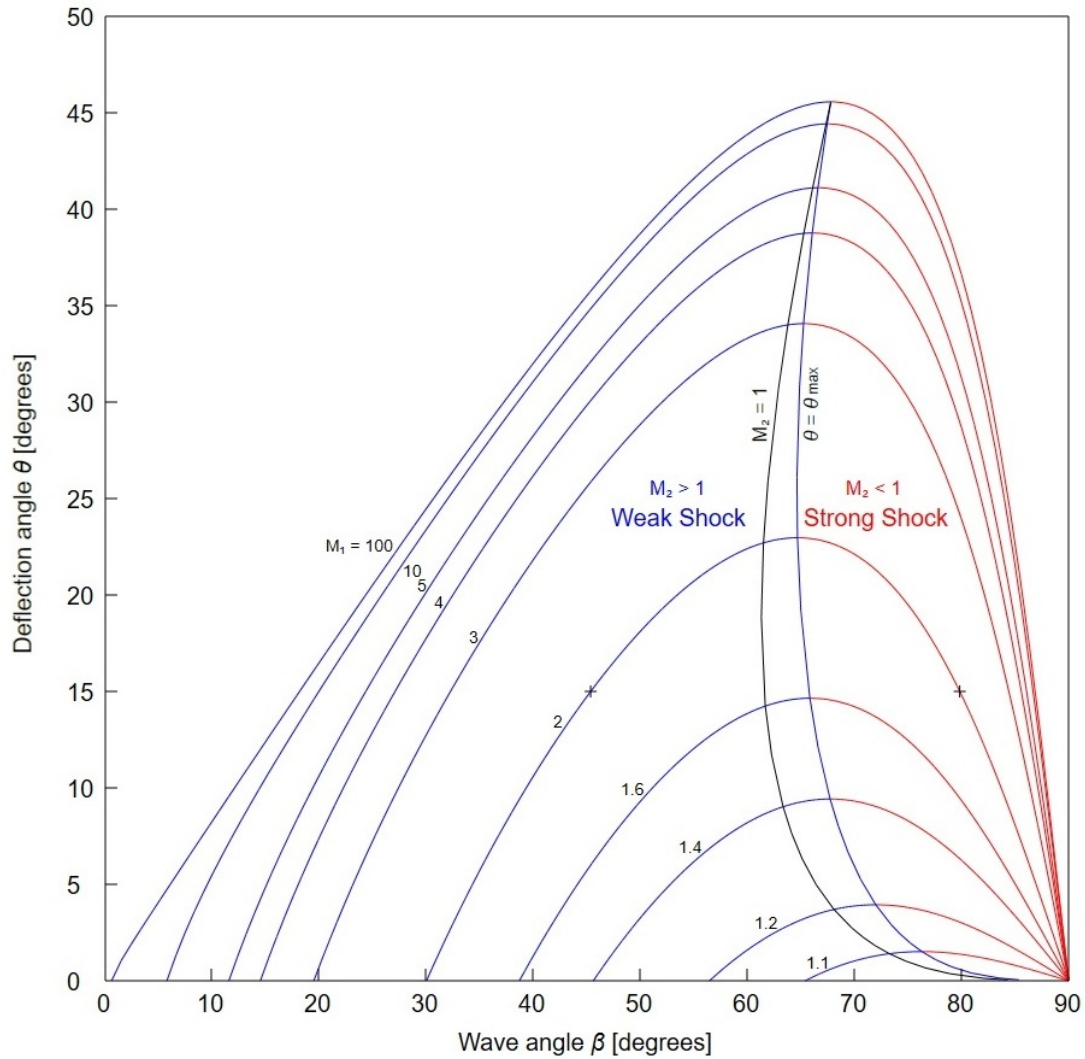


Figure 5 Weak and strong shocks for the solutions in Figure 3

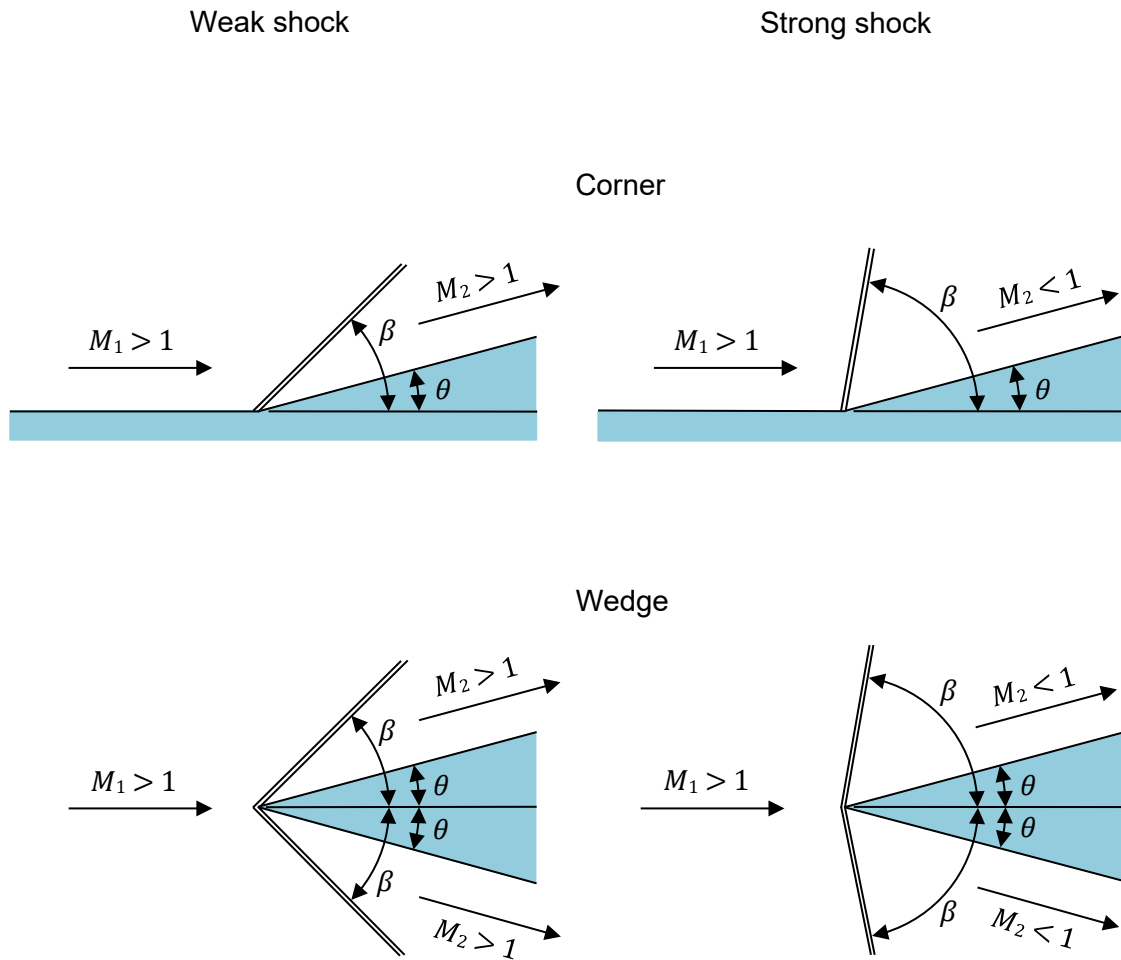


Figure 6 Message if $\theta > \theta_{max}$

Oblique Shock

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Upstream of shock

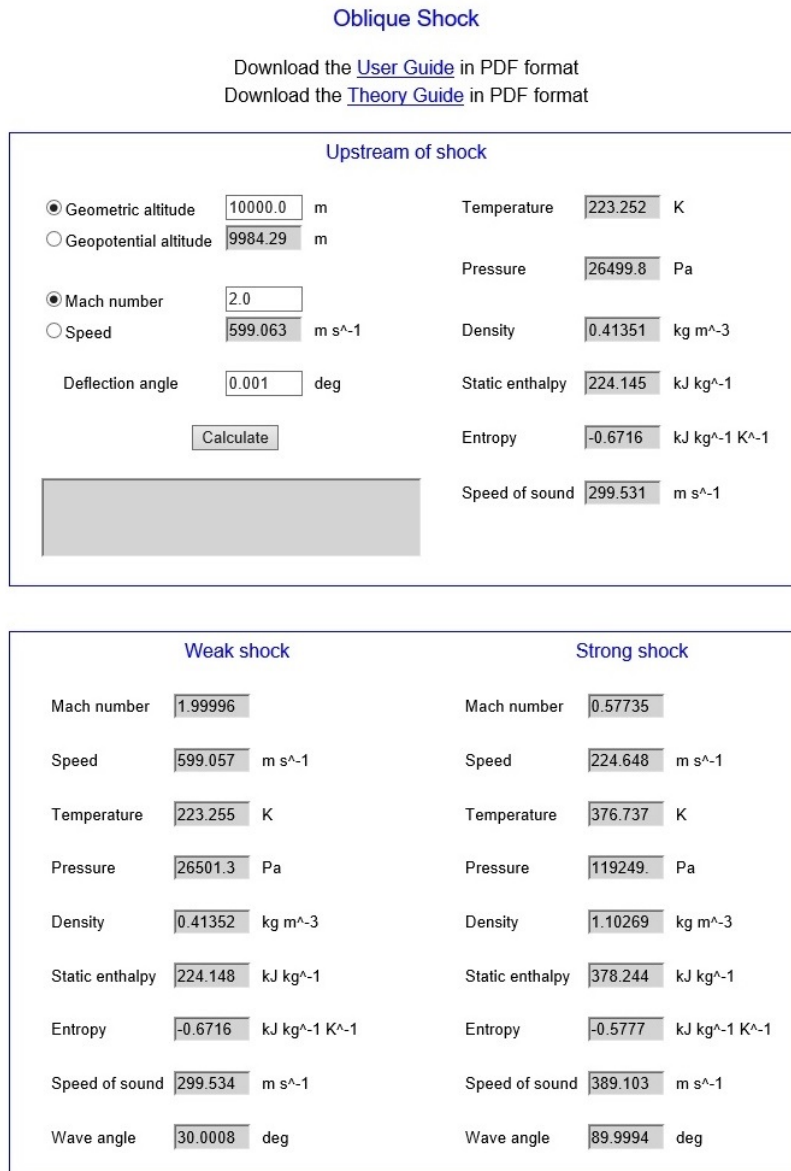
<input checked="" type="radio"/> Geometric altitude	<input type="text" value="10000.0"/> m	Temperature	<input type="text"/> K
<input type="radio"/> Geopotential altitude	<input type="text"/> m	Pressure	<input type="text"/> Pa
<input checked="" type="radio"/> Mach number	<input type="text" value="2.0"/>	Density	<input type="text"/> kg m ⁻³
<input type="radio"/> Speed	<input type="text"/> m s ⁻¹	Static enthalpy	<input type="text"/> kJ kg ⁻¹
Deflection angle	<input type="text" value="25.0"/> deg	Entropy	<input type="text"/> kJ kg ⁻¹ K ⁻¹
<input type="button" value="Calculate"/>		Speed of sound	<input type="text"/> m s ⁻¹

There is no solution. The shock wave is detached. Either increase the Mach number or reduce the deflection angle to produce an oblique shock wave.

Weak shock	Strong shock
Mach number <input type="text"/>	Mach number <input type="text"/>
Speed <input type="text"/> m s ⁻¹	Speed <input type="text"/> m s ⁻¹
Temperature <input type="text"/> K	Temperature <input type="text"/> K
Pressure <input type="text"/> Pa	Pressure <input type="text"/> Pa
Density <input type="text"/> kg m ⁻³	Density <input type="text"/> kg m ⁻³
Static enthalpy <input type="text"/> kJ kg ⁻¹	Static enthalpy <input type="text"/> kJ kg ⁻¹
Entropy <input type="text"/> kJ kg ⁻¹ K ⁻¹	Entropy <input type="text"/> kJ kg ⁻¹ K ⁻¹
Speed of sound <input type="text"/> m s ⁻¹	Speed of sound <input type="text"/> m s ⁻¹
Wave angle <input type="text"/> deg	Wave angle <input type="text"/> deg

The user may not enter a deflection angle θ less than or equal to zero. However, if the user enters a very small value, say 0.001° , then he/she will effectively obtain the two solutions along the axis $\theta = 0$, as shown in Figure 7. The weak shock is vanishingly weak, with essentially no change in the flow properties across the shock. The wave angle β for the weak shock is then called the Mach angle μ . The wave angle for the strong shock is essentially 90° . Unlike the weak shock, the strong shock continues to produce a change in the flow properties.

Figure 7 Properties for θ very close to zero



3 Further reading

The equations of the oblique shock and the assumptions on which they are based are set out in Ref. [2], which can be downloaded from the Atkinson Science web site, <https://atkinsonscience.co.uk>. The derivation of the equations can be found in text books on compressible flows, such as Refs. [3] and [4]. Not all text books describe a method of determining the wave angle β from the upstream Mach number M_1 and deflection angle θ . The method used in the web application is explained in Ref. [2].

The Atkinson Science web site has a free-to-use web application for determining the properties of the International Standard Atmosphere, given the geometric or geopotential altitude. There is also a user guide and a theory guide that can be downloaded.

4 References

1. *International Standard Atmosphere*, ISO 2533:1975, International Standards Organisation, 1975.
2. K. N. Atkinson, *Oblique Shock Web Application, Theory Guide*, Atkinson Science Limited, 21 September 2020 (download from <https://atkinsonscience.co.uk>).
3. J. D. Anderson, *Modern Compressible Flow with Historical Perspective*, 3rd Ed., McGraw-Hill, 2004.
4. H. W. Liepmann and A. Roshko, *Elements of Gasdynamics*, Dover, 2001.